Singaporean Journal of Scientific Research
新加坡科学研究期刊 新加坡科学研究期刊 新加坡科学研究期刊 新加坡科学研究期刊 新加坡科学研究期刊

Singaporean Journal of Scientific Research(SJSR)
An International Journal (AMIJ)
Vol.10.No.1 2018,Pp.10-17
available at :www.sjsronline.com
Paper Received : 12-01-2018
Paper Accepted: 15-03-2018
Paper Reviewed by: 1.Prof. Cheng Yu 2. Dr.M. Akshay Kumar
Editor : Dr. R. Akilesh Mathur

An Innovative Study on Odd and Even Vertex Magic Labeling of Graphs

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1. INTRODUCTION

Graph theory is the fast growing area of mathematics because of its inherent simplicity. Graph theory may be said to have its begining in 1736 . When Euler considered the general case of the Konigsberg bridge problem . Since, then graph theory has developed into an extensive and popular branch of mathematics which has been applied to many problem in mathematics and other scientific areas.

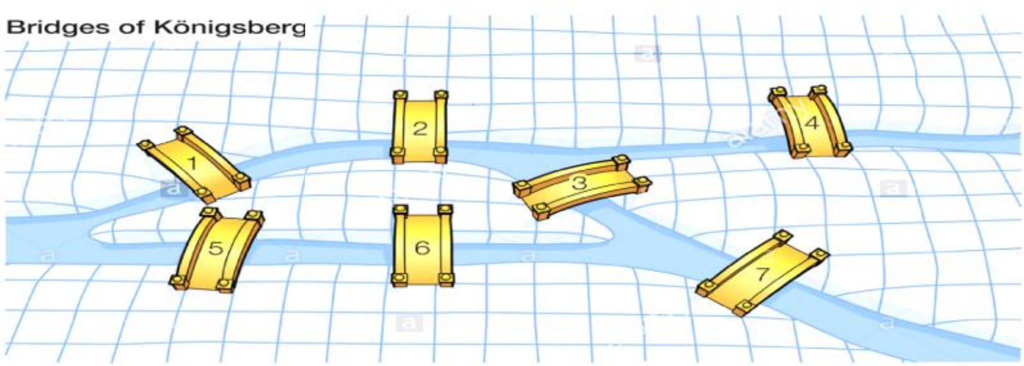

Figure 1: View of Bridges and Konigsberg

Graph theory has a wide range of applications in computer science, sociology, bio medicine, medicine etc . Many problem like racing fault tolerance coding embedding etc can easily be modeled and explained through graph theoretical terminologies. Graph theory has had an usual development problems involving Graphs first appeared in the mathematical puzzles. Today graph theory is one of the most flourishing branches of modern algebra with wide applications to combinatorial problems and to classical algebraic problems.

Graph labeling serves as a frontier between number theory and structure of graphs .Graph labeling have enormous applications within mathematics as well as to several areas of computer science and communication network.

2. Preliminaries

In this chapter we collect the basic definitions which are needed for the subsequent chapter. For the notations and terminology we refer to J.A. Bondy and U.S.R. Murthy.

Definition 1:1

A Graph G is a finite non empty set V (G) of elements called vertices (or points) and a set E (G), of unordered pairs of distinct elements of V (G) called edges (or lines). V (G) and E (G) are called vertex set and edge set respectively.

Definition 1:2

A graph without parallel edges and loops is called a simple graph.

Definition 1:3

Let G = (V, E) be a graph and let W V and F E then H = (W, F) is called a sub graph of G.

Definition 1:4

A spanning sub graph is a sub graph containing all the vertices of G.

Definition 1:5

A graph H = {V1 ,X1}is called a subgraph of G ={V,X}. If V1≤ V and X1≤ X.If H is a subgraph G. we say not G is a super graph of H.H is called a spanning subgraph of G. If V1=V. H is called induced subgraph of G.

Definition 1:6

The degree of a vertex V in a graph G is the number of lines incident with V. The degree of d v is denoted by d G (V) or deg V or simply d (V).

Definition 1:7

A vertex having no incident edge is called isolated vertex. Any vertex of degree one is called a pendant vertex.

Definition 1:8

If more than one line joining two vertices are allowed the resulting object is called multi graph.

Definition 1:9

A graph in which any two distinct points are adjacent is called complete graph.

The complete graph with P points is denoted by K p.

Definition 1:10

Graph whose edge set is empty is called a null graph or a totally disconnected graph.

Definition 1:11

A graph G = (V, E) is called a bipartite graph if its vertex set V can be partitioned into two subsets V1 and V2 such that each edge of E has one end vertex in V1 and another in V2 (V1,V2) is referred to as a bipartite of G.

Definition 1:12

If all the points of G have the same degree r than d(G) = D(G) = r is called regular graph.

Definition 1:13

A graph G = (V, E) is said to be connected graph if any two vertices in G are connected by a path.

Definition 1:14

The maximal connected subgraphs of G are called its components.

Definition 1:15

A subset S of V is called an independent set of G if no two vertices of S are adjacent in G.

Definition 1:16

A walk is defined as a finite alternating sequence of vertices and edges which begins and ends with vertices such that no edge appears more than once in a sequence such a sequence is called a walk or trail in G.

Definition 1:17

A walk that begins and ends at the same vertex is called a closed walk. A walk that is not closed is called an open walk.

Definition 1:18

A closed walk v0, v 1, v2….v n= vn in which n 3 and v 0, v 1….v n-1 are distinct is called a cycle of length n. It is denoted by C n.

Definition 1:19

A Tree is a connected acyclic graph.

Definition 1:20

A cut vertex is a vertex that when removed from a graph creates more components than previously in the graph.

Definition 1:21

A connected non-trivial graph having no cut point is a block.

Definition 1:22

If the vertices are assigned values subject to certain conditions then it is known as Graph labeling.

Definition 1:23

A magic graph is a graph whose edges are labeled by positive integers, so that the sum over the edges incident with any vertex is the same, independent of the choice of vertex, or it is a graph that has such a labeling.

Definition 1:24

A vertex magic total labeling of a graph with V vertices and E edges is denoted as a one to one map taking the vertices and edges onto the integers 12 v + e with the property that the sum of the label on a vertex and the labels on its incident edges is a constant independent of the choice of vertex.

3. Odd Vertex Magic Total Labeling of Graphs

Theorem 3.1

rC_s is an odd vertex magic iff r and s are odd.

Proof:

Assume that rC_s is odd vertex magic.

Number of vertices of rC_s is rs.

Number of edges of rC_s is rs.

Then by Theorem 2.3, k = 3rs + 2. For any odd vertex magic total labeling f, k = f(u) $\sum_{v\in N(u)} f(uv) \forall u \in V$. since any vertex of rCs is adjacent to only two edges, $f(u)+f(e_1) + f(e_2) = k$, where e_1 and e_2 are the edges adjacent to u.

Here f(u) is an odd number and $f(e_1)$ and $f(e_2)$ are even numbers.

Therefore k = 3 rs + 2 is odd iff r and s are odd

Let r and s be odd integers.

Assume that the graph rc_s has the vertex set $V = V_1 \cup V_2 \cup V_3 \cup ,.... \cup V_r$, where $v_i = \{V_i^1, V_i^2, V_i^3, ...V_i^s,\}$ and the edge set $E = E_1 \cup E_2 \cup E_3 \cup ,......... \cup E_r$ where $E_i = \{e_i^1, e_i^2, e_i^3, ...e_i^s,\}$, and $e_i^j = v_i v_i^{(j+1)}$ for $1 \le i \le r, 1 \le j \le s-1, e_i^s = v_i^s v_i^1$

Define $f(V \cup E) = \{1, 2, 3, ..., 2rs\}$ as follows:

$$f(v_i^1) = 2(rs - i) + 1, \quad i = 1, 2, ..., r$$

$$f(v_i^2) = \begin{cases} 2(rs - 2r + 2i) - 1, & 1 \le i \le \frac{(r-1)}{2} \\ 2(rs - 3r + 2i) - 1, & \frac{(r+1)}{2} \le i \le r. \end{cases}$$

For j = 3, 4, 5, ..., s

$$f(v_i^j) = \begin{cases} 2rs - (2j-1)r - 2i, & 1 \le i \le \frac{(r-1)}{2} \\ 2rs - (2j-3)r - 2i, & \frac{(r+1)}{2} \le i \le r. \end{cases}$$

$$f(e_i^1) = \begin{cases} r - 2i + 1, & 1 \le i \le \frac{(r-1)}{2} \\ 3r - 2i + 1, & \frac{(r+1)}{2} \le i \le r. \end{cases}$$

j = 2, 4, 6, ..., s – 1

$$f(e_i^s) = rs - 2i + 2 + (j+1)r, \quad 1 \le i \le r.$$

j = 3, 5, 7, ..., s

$$f(e_i^j) = \begin{cases} 4i + (j-1)r, & 1 \le i \le \frac{(r-1)}{2} \\ 4i + (j-3)r, & \frac{(r+1)}{2} \le i \le r \end{cases}$$

It is clearly verified that f is odd vertex magic labeling of rC_s with the magic constant k = 3rs + 2.

Example 3.2

An odd vertex magic labeling of $5C_3$ is given in figure.

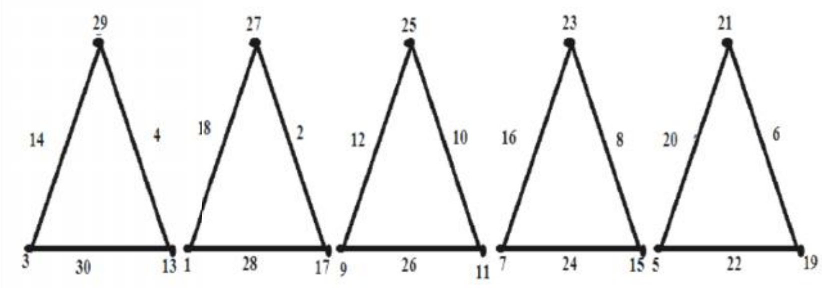

Figure 3 n = 15,m = 15, k = 47

Theorem 3.3

(s, t) − kite graph admits an odd vertex magic labeling iff s + t is odd.

Proof.

Let G be a (s, t) − kite graph.

Let the vertex set

$V = \{v_1, v_2, v_3, ..., v_s\} \cup \{u_1, u_2, u_3, ..., u_t\}$ and the edge set

$E = \{e_i = v_iv_{i+1}, e_n = v_sv_1, 1 \le i \le s-1\} \cup \{x_i = u_iv_i+1, x_t = u_tv_1 : 1 \le i \le t-1\}$.

Hence n = m = s + t. Suppose G admits an odd vertex magic total labeling f with a vertex magic constant k.

Then nk = $\sum_{u\in V} f(u) + 2\sum_{e\in E} f(e).$ $\quad Hence k = 3(s+t)+2$

To prove that s + t is odd

Suppose s + t is even.

Therefore k is even.

If u is pendent vertex of (s, t) kite, them f(u) + f(u v) = k, which is a contradiction.

Since f(u) is odd and f(u v) is even. Therefore s + t is odd.

Conversely : Suppose s + t is odd.

Hence either s or t is odd. We consider two cases.

Case (i) : s is odd and t is even,

Define f : V ∪ E → {1,2,3,..............., 2s+2t} as follows, For $1 \le i \le s$,

$$f(e_i) = \begin{cases} t+i+1, & if \quad i \quad is \quad odd \\ 2t+s+1+i, & if \quad i \quad is \quad even \end{cases}$$

For $1 \le i \le t$,

$$f(x_i) = \begin{cases} t+s+i+2, & if \quad i \quad is \quad odd \\ i, & if \quad i \quad is \quad even \end{cases}$$

The vertex labeling as follows :

$$f(v_i) = 2s - 2i + 1, \qquad if \quad 1 \le i \le s$$

$$f(u_i) = 2s + 2t + 1 - 2i, \qquad if \quad 1 \le i \le t$$

It could be easily verified that f is an vertex magic labeling of G with k = 3(s + t) + 2.

Case (ii): S is even and t is odd. We consider two subcases.

Subcase (i): t > s. For $1 \le i \le s$

$$f(e_i) = \begin{cases} 2s-i+1, & if \quad i \quad is \quad odd \\ 3s+t-i+1, & if \quad i \quad is \quad even \end{cases}$$

and for $1 \le i \le t$,

$$f(x_i) = \begin{cases} 3s+t+i, & if \quad i = 1,3,5,...,t-s \\ i-(t-s), & if \quad i = t-s+2, t-s+4, ...,t \\ 2s+i, & if \quad i \quad is \quad even \end{cases}$$

The vertex labeling are as follows,

$$f(u_1) = 2t + 1$$

$$f(u_i) = \begin{cases} 2t - 2i - 2s + 3, & if \quad 2 \le i \le t - s + 1 \\ 4t - 2i + 3, & if \quad t - s + 2 \le i \le t \end{cases}$$

$$f(v_i) = 2t + 2i - 2s - 1 \quad for \quad 1 \le i \le s$$

It could be verified that f is an odd vertex magic total labeling of G with k = 3(s + t) + 2.

Subcase (ii) t ≤ s. For 1 ≤ i ≤ s

$$f(c_i) = \begin{cases} t - i, & if \quad i = 1, 3, ..., t - 2 \\ 3t + 2s - i, & if \quad i = t, t + 2, ..., s - 1 \\ 2t + s - i, & if \quad i = 2, 4, ..., s \end{cases}$$

$$f(x_i) = \begin{cases} 2t - i, & if \quad i = 2, 4, ..., t - 1, 1 \le i \le t \\ 3t + s - i, & if \quad i = 1, 3, ..., t \quad 1 \le i \le t \end{cases}$$

The vertex labeling are as follows:

$$f(v_1) = 2s - 2t + 3$$

$$f(v_i) = \begin{cases} 2s + 2i + 1, & if \quad 2 \le i \le t - 1 \\ 2i - 2t + 1, & if \quad t \le i \le s \end{cases}$$

$$f(u_1) = 2s + 3$$

$$f(u_i) = 2s - 2t + 2i + 1, \quad if \quad 2 \le i \le t,$$

It can be clearly proved that f is an odd vertex magic total labeling with k = 3(s + t) + 2

4. Conclusion

Graph Theory deals with the study of problems involving discrete arrangement objects, where concern is not with the internal properties of the objects but the relationship among them. A magic graph is a graph whose vertex are labeled and edge are labeled by integers, so that the sum over the edges incident with any vertex is the same, independent of the choice of vertex, or it is a graph that has such a labeling. There are a great many variations on the concept of magic labeling of a graph. There is much variation in terminology as well.

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